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Radial excitations of low–lying baryons and the Z^+ penta–quark

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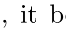
Abstract. Within an extended Skyrme soliton model for baryons the interplay between the collective radial motion and the SU(3)–flavor–rotations is investigated. The coupling between these modes is mediated by flavor symmetry breaking. Collective coordinates which describe the corresponding large amplitude fluctuations are introduced and treated canonically. When diagonalizing the resulting Hamiltonian flavor symmetry breaking is fully taken into consideration. As eigenstates not only the low–lying $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons but also their radial excitations are obtained and compared to the empirical data. In particular the relevance of radial excitations for the penta–quark baryon Z^+ ($Y = 2$, $I = 0$, $J^\pi = \frac{1}{2}^+$) is discussed. In this approach its mass is predicted to be 1.58GeV. Furthermore the widths for various hadronic decays are estimated which, for example, yields $\Gamma(Z^+ \to NK) \sim 100$MeV for the only permissible decay process of the Z^+.

PACS. 12.39.Dc Skyrmions – 14.20.Gk Baryon resonances with $S = 0$ – 14.20.Jn. Hyperons

1 Introduction

Recently there has been renewed interest in baryon states which cannot be described as simple bound states of three quarks [1–3]. One of the most prominent examples is the so–called Z^+ which possesses the spin and isospin quantum numbers of the Λ hyperon, however, it carries hypercharge $Y = 2$. When extending chiral soliton models to flavor SU(3) [4,5] such states come about quite naturally as they are members of higher dimensional representations which do not have counterparts of equal quantum numbers in the octet or decuplet.

In the context of chiral soliton models these higher dimensional representations have gained most of their recognition from the investigation of flavor symmetry breaking. Besides such *exotic* states as the Z^+, the higher dimensional representations also contain states which have the spin and flavor quantum numbers of the low–lying $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons. One easily recognizes that flavor symmetry breaking couples states which belong to different SU(3) representations but otherwise have identical quantum numbers. Hence these states mix with the octet and decuplet baryons as the higher dimensional representations provide a basis to obtain the exact eigenstates of the full collective Hamiltonian [6]. As a consequence the nucleon is no longer a pure octet state but also contains sizable admixture of the corresponding members in the $\overline{\mathbf{10}}$ and $\mathbf{27}$ dimensional representations [7,8]. Permissible representations are those which contain a non–strange baryon state with identical spin and isospin [9].

Presumably the lightest state which does not have a counterpart of equal quantum numbers in the octet or decuplet is the above mentioned Z^+. The lowest dimensional representation containing a state with the quantum numbers of the Z^+ is the $\overline{\mathbf{10}}$. When glancing at the Young tableau of the $\overline{\mathbf{10}}$, , it becomes immediately clear that such states are not simple bound states containing only three quarks. Rather they have to be interpreted as a quark–antiquark pair coupled to a three quark state. Such objects are commonly called penta–quarks. In [1, 2] quantitative calculations for the mass of the Z^+ were performed within the Skyrme model [10–12]. From Fig. 2 of [1] one deduces that the Z^+ should be about 0.7GeV heavier than the nucleon. This is not too different from the estimate of [3] where a mass difference of 0.59GeV with respect to the nucleon was predicted. However, the latter prediction is just 100MeV above the threshold for the only accessible decay process, $Z^+ \to NK$. While the result obtained in [1] stems from a self–contained model calculation the authors of [3] collected almost[1] all contributions up to linear order in flavor symmetry breaking to the collective Hamiltonian and the baryon wave–functions which are consistent with the general transformation properties of these objects in flavor space. The associated constants of proportionality were determined from data for the established baryons. In particular (up to flavor symmetry breaking effects) the $N(1710)$ was identified with the nucleon state in the $\overline{\mathbf{10}}$ representation in order to fix the

[1] For example, the admixture of the states of the $\mathbf{27}$–plet to the octet as well as the coupling between the Z^+–type states in the $\overline{\mathbf{10}}$ and $\overline{\mathbf{35}}$ representations were omitted

mass difference of the states within that representation to the octet baryons. This treatment is not without ambiguities because not all baryons are rotational excitations. In such a picture one first wonders about the role of the Roper (1440) resonance which in Skyrmion models often is identified as the radial (or breathing) $\hbar\omega$ excitation of the nucleon [13–15]. Secondly it is then natural to consider the $N(1710)$ resonance as the corresponding $2\hbar\omega$ excitation. This scenario leads to the obvious question whether there is an interplay between radial and rotational excitations and especially to what extent this interplay effects the predictions for the Z^+. The coupling between these two types of excitations is mediated by flavor symmetry breaking. For the ordinary baryons this interplay has already been discussed some time ago [16,17]. It is the purpose of the present study to extend this approach to the Z^+ for which the radial motion has not been considered previously.

In Sect. 2 the simultaneous treatment of collective coordinates for radial and rotational motion of the soliton will be discussed for a special chiral model. Also the resulting baryon spectrum will be compared to the empirical data. In Sect. 3 the widths for various hadronic decay modes of excited baryons will be estimated. Section 4 will serve to summarize the present study.

2 Breathing mode approach in flavor SU(3)

In this section we will describe the treatment of large amplitude fluctuations for radial and rotational degrees of freedom in a three flavor soliton model. The simplest model within which such a study can be carried out is the Skyrme model with only pseudoscalar fields. Unfortunately, the breathing mode approach to this model does not adequately reproduce the mass differences in baryon spectrum [16]. For the present investigation we will therefore employ an extended version of the Skyrme model by supplementing it with a scalar field as motivated by the trace anomaly of QCD. Although the breathing mode approach to this model reasonably describes the baryon spectrum as well as various static baryon properties [17] it seems somewhat unmotivated. A more natural model choice would rather employ vector meson [18] or chiral quark [20] models. The reason being that these mesonic degrees of freedom need to be included in order to obtain non–vanishing neutron–proton mass differences as well as a finite axial singlet current matrix element of the nucleon [21]. In these models meson fields, which vanish classically, are induced by the collective rotation $A(t)$ in eq (2.8). As it is yet unknown how to treat the breathing mode in the presence of these induced fields the model may be considered as an effective parameterization of massive meson fields. A particular difficulty with these induced fields is that the corresponding stationary conditions must separately be solved for every value of the scaling coordinate x in order to maintain the correct normalization of the Noether currents. In vector meson models this problem occurs already on the classical level for the time component of the ω field because its stationary condition actually is a constraint which guarantees the positivity of the classical energy functional. The incorporation of the breathing coordinate in an SU(2) vector meson model was attempted in [19], however, the above mentioned subtleties were ignored. In non–topological chiral quark soliton models their non–confining character may cause additional problems when treating the breathing mode dynamically because a transition to the trivial meson configuration may occur [15].

To be specific we will follow the treatment of [17] where the soliton model not only contains the pseudoscalar mesons ϕ^a but also an effective scalar meson field $H = \langle H\rangle \exp(4\sigma)$ which is introduced to mock up the QCD anomaly [22] for the dilatation current

$$-\partial_\mu D^\mu = H + \sum_i m_i \bar{\Psi}_i \Psi_i \quad \text{where}$$
$$H = -\frac{\beta(g)}{g} G^a_{\mu\nu} G^{a\mu\nu}. \qquad (2.1)$$

The vacuum expectation value $\langle H\rangle \sim (0.30 - 0.35\mathrm{GeV})^4$ can be extracted from sum rule estimates for the gluon condensate [23]. Eventually the fluctuating field σ may be identified as a scalar glueball. The effective mesonic action reads

$$\Gamma = \int d^4x \left(\mathcal{L}_0 + \mathcal{L}_{\rm SB}\right) + \Gamma_{\rm WZ} \,. \qquad (2.2)$$

The flavor symmetric part involves both the chiral field[2] $U = \exp(i\lambda^a \phi^a / f_a)$ as well as the scalar gluonic fluctuation σ

$$\begin{aligned}\mathcal{L}_0 = &-\frac{f_\pi^2}{4} \mathrm{e}^{2\sigma} \mathrm{tr}\left(\alpha_\mu \alpha^\mu\right) + \frac{1}{32e^2} \mathrm{tr}\left(\left[\alpha_\mu, \alpha_\nu\right]^2\right) \\ &+ \frac{1}{2}\Gamma_0^2 \ \mathrm{e}^{2\sigma} \ \partial_\mu \sigma \partial^\mu \sigma + \mathrm{e}^{4\sigma} \\ &\times \left\{\frac{1}{4}\left[\langle H\rangle - 6\left(2\delta' + \delta''\right)\right] - \sigma \langle H\rangle\right\} \end{aligned} \qquad (2.3)$$

with $\alpha_\mu = \partial_\mu U U^\dagger$. Assuming the canonical dimensions $d(U) = 0$ and $d(H) = 4$ it is straightforward to verify that (2.3) yields the anomaly Eq. (2.1) for $m_i = 0$. The terms which lift the degeneracy between mesons of different strangeness are comprised in

$$\begin{aligned}\mathcal{L}_{\rm SB} = \mathrm{tr}\Big\{ &\left(\beta' \hat{T} + \beta'' \hat{S}\right) \mathrm{e}^{2\sigma} \partial_\mu U \partial^\mu U^\dagger U \\ &+ \left(\delta' \hat{T} + \delta'' \hat{S}\right) \mathrm{e}^{3\sigma} U + \mathrm{h.c.} \Big\} \,, \end{aligned} \qquad (2.4)$$

where the flavor projectors $\hat{T} = \mathrm{diag}(1,1,0)$ and $\hat{S} = \mathrm{diag}(0,0,1)$ have been introduced. Using a sigma–model interpretation of the chiral field the coupling of the scalar field in $\mathcal{L}_{\rm SB}$ is such as to reproduce the explicit breaking in the anomaly eq. (2.1) [24]. The major impact of the scalar field emerges through the factor $\mathrm{e}^{3\sigma}$ in the mass term of

[2] Here the normalization coefficients f_a refer to the pseudoscalar decay constants

the symmetry breaking piece (2.4). As will be discussed later, this mitigates the symmetry breaking effects in the baryon sector. This factor is special to the model with the trace anomaly included since it properly accounts for the explicit breaking of the dilatation current (2.1) as the quark bilinear $\bar{\Psi}_i\Psi_i$ has canonical mass dimension three.

The various parameters in (2.3) and (2.4) are determined from the masses and decay constants of the pseudoscalar mesons:

$$\beta' \approx 26.4\mathrm{MeV}^2,\ \beta'' \approx 985\mathrm{MeV}^2,$$
$$\delta' \approx 4.15\times 10^{-5}\mathrm{GeV}^2,\ \delta'' \approx 1.55\times 10^{-3}\mathrm{GeV}^4 \quad (2.5)$$

Then the only free parameters of the model are the Skyrme constant e and the glueball mass,

$$m_\sigma^2 = \frac{4\langle H\rangle + 6(2\delta'+\delta'')}{\Gamma_0^2}\ . \quad (2.6)$$

As in [17] we will use $m_\sigma \approx 1.25$GeV. Finally the scale invariant Wess–Zumino term [25] is most conveniently presented by introducing the one–form $\alpha = \alpha_\mu dx^\mu$,

$$\Gamma_{\rm WZ} = \frac{iN_c}{240\pi^2}\int \mathrm{tr}(\alpha^5)\ . \quad (2.7)$$

The above described model possesses a static soliton solution $U_0(\boldsymbol{r}) = \exp[i\boldsymbol{\tau}\cdot\hat{\boldsymbol{r}}F(r)]$, $\sigma(\boldsymbol{r}) = \sigma_0(r)$ which is characterized by the two radial functions $F(r)$ and $\sigma_0(r)$ [22,26]. Except of unit baryon number this configuration does not carry baryonic quantum numbers such as spin or isospin. Baryon states are commonly generated by canonical quantization of the collective coordinates which are introduced to describe large amplitude fluctuations. Apparently these are the rotations in coordinate and flavor spaces which are (up to flavor symmetry breaking) zero modes of the soliton. Due to the hedgehog structure of the soliton these rotations are equivalent. In addition the energy surface associated with scale or breathing transformations of the Skyrmion is known to be flat, at least in a large vicinity of the stationary point [13,14]. For this reason it is suggestive to also introduce a collective coordinate for the soliton extension. Then the unknown time–dependent solution to the Euler equations is approximated by

$$U(\boldsymbol{r},t) = A(t)U_0\left(\mu(t)\boldsymbol{r}\right)A^\dagger(t) \quad \text{and}$$
$$\sigma(\boldsymbol{r},t) = \sigma_0\left(\mu(t)r\right)\ . \quad (2.8)$$

Substituting this parameterization into the action (2.2) yields the Lagrangian for the collective coordinates $A(t)$ as well as $x(t) = [\mu(t)]^{-3/2}$

$$\begin{aligned} L(x,\dot{x},A,\dot{A}) = &\frac{4}{9}\left(a_1 + a_2 x^{-\frac{4}{3}}\right)\dot{x}^2 - \left(b_1 x^{\frac{2}{3}} + b_2 x^{-\frac{2}{3}} + b_3 x^2\right) \\ &+ \frac{1}{2}\left(\alpha_1 x^2 + \alpha_2 x^{\frac{2}{3}}\right)\sum_{a=1}^{3}\Omega_a^2 \\ &+ \frac{1}{2}\left(\beta_1 x^2 + \beta_2 x^{\frac{2}{3}}\right)\sum_{a=4}^{7}\Omega_a^2 + \frac{\sqrt{3}}{2}\Omega_8 \\ &- \left(s_1 x^2 + s_2 x^{\frac{2}{3}} + \frac{4}{9}s_3\dot{x}^2\right)(1-D_{88})\ . \end{aligned} \quad (2.9)$$

Here the angular velocities $A^\dagger\dot{A} = (i/2)\sum_{a=1}^{8}\lambda_a\Omega_a$ as well as the adjoint representation $D_{ab} = (1/2)\mathrm{tr}(\lambda_a A\lambda_b A^\dagger)$ have been introduced. A term linear in $\dot{x}$, which would originate from flavor symmetry breaking terms, has been omitted because the matrix elements of the associated $SU(3)$ operators vanishes when properly accounting for Hermiticity in the process of quantization [27]. The expressions for the constants $a_1,\ldots,s_3$ as functionals of the chiral angle as well as their numerical values may be extracted from [17]. The term involving s_3 causes major difficulties in the process of quantization. This contribution to L stems from the derivative type symmetry breaker in (2.4) whose influence in the soliton sector is known to be small[3]. In addition, by replacing the collective function $1-D_{88}$ with a constant of order unity the effects of this term have been estimated to be only a few percent, *cf.* appendix B of [16]. Hence the s_3 term may safely be omitted.

The baryon states corresponding to the Lagrangian (2.9) are obtained in a two–step procedure. In the first step flavor symmetry breaking is ignored. For convenience one furthermore defines

$$\begin{aligned} m &= m(x) = \frac{8}{9}\left(a_1 + a_2 x^{-\frac{4}{3}}\right)\ , \\ b &= b(x) = b_1 x^{\frac{2}{3}} + b_2 x^{-\frac{2}{3}} + b_3 x^2\ , \\ \alpha &= \alpha(x) = \alpha_1 x^2 + \alpha_2 x^{\frac{2}{3}}\ , \\ \beta &= \beta(x) = \beta_1 x^2 + \beta_2 x^{\frac{2}{3}} \end{aligned}$$

and

$$s = s(x) = s_1 x^2 + s_2 x^{\frac{2}{3}}\ . \quad (2.10)$$

Then the flavor symmetric part of the collective Hamiltonian

$$\begin{aligned} H = &-\frac{1}{2\sqrt{m\alpha^3\beta^4}}\frac{\partial}{\partial x}\sqrt{\frac{\alpha^3\beta^4}{m}}\frac{\partial}{\partial x} + b \\ &+\left(\frac{1}{2\alpha} - \frac{1}{2\beta}\right)J(J+1) + \frac{1}{2\beta}C_2(\mu) - \frac{3}{8\beta} + s \end{aligned} \quad (2.11)$$

is diagonalized for a definite $SU(3)$ representation μ. Due to the hedgehog structure of the static configuration U_0 and σ_0, the allowed representations must contain at least one state with identical spin and isospin. In addition, this state must have vanishing strangeness [4,5]. For definiteness we denote the eigenvalues of (2.11) by $\mathcal{E}_{\mu,n_\mu}$ and the corresponding eigenstates by $|\mu,n_\mu\rangle$, where n_μ labels the radial excitations. Actually the eigenstates factorize $|\mu,n_\mu\rangle = |\mu\rangle|n_\mu\rangle$. In this language the nucleon corresponds to $|\boldsymbol{8},1\rangle$ while the first radially excited state, which

[3] In numerical calculations the direct contributions of this term are small. Nevertheless it is important because it has significant indirect influence since it provides the origin for different decay constants, $f_K \approx 1.2 f_\pi$. Compared to the unphysical case $f_\pi = f_K$ the mass type symmetry breaker increases by about 50% because $\delta'' = (2f_K^2 m_K^2 - f_\pi^2 m_\pi^2)/4$. The δ'' term is contained in s_1

Table 1. The mass differences with respect to the nucleon (939MeV) of the eigenstates of the Hamiltonian (2.12). Experimental data are taken from [29], if available. It should be remarked that even single–star resonances have been included. The notation for the states appearing in this table is defined in eq (2.13). All numbers are in MeV

B	$m=0$			$m=1$			$m=2$		
	e=5.0	e=5.5	expt.	e=5.0	e=5.5	expt.	e=5.0	e=5.5	expt.
N		Input		413	445	501	836	869	771
Λ	175	173	177	657	688	661	1081	1129	871
Σ	284	284	254	694	722	721	1068	1096	838
Ξ	382	380	379	941	971	—	1515	1324	—
Δ	258	276	293	640	680	661	974	1010	981
Σ^*	445	460	446	841	878	901	1112	1148	1141
Ξ^*	604	617	591	1036	1068	—	1232	1269	—
Ω	730	745	733	1343	1386	—	1663	1719	—

is commonly identified with the Roper (1440) resonance, would be $|\mathbf{8},2\rangle$. Of course, we are interested in the role of states like $|\overline{\mathbf{10}},n_{\overline{\mathbf{10}}}\rangle$ since in particular this tower contains the state with the quantum numbers of the Z^+. In the second step the symmetry breaking part will be taken into account. This is done by employing the states $|\mu,n_\mu\rangle$ as a basis to diagonalize the complete Hamiltonian matrix

$$H_{\mu,n_\mu;\mu',n'_{\mu'}} = \mathcal{E}_{\mu,n_\mu}\delta_{\mu,\mu'}\delta_{n_\mu,n'_{\mu'}} -\langle\mu|D_{88}|\mu'\rangle\langle n_\mu|s(x)|n'_{\mu'}\rangle \ . \quad (2.12)$$

The flavor part of these matrix elements is computed using $SU(3)$ Clebsch–Gordon coefficients[4] while the radial part is calculated numerically using the appropriate eigenstates of (2.11). Of course, this can be done for each isospin multiplet separately, *i.e.* flavor quantum numbers are not mixed. The physical baryon states $|B,m\rangle$ are finally expressed as linear combinations of the eigenstates of the symmetric part

$$|B,m\rangle = \sum_{\mu,n_\mu} C^{(B,m)}_{\mu,n_\mu}|\mu,n_\mu\rangle \ . \quad (2.13)$$

The corresponding eigenenergies are denoted by $E_{B,m}$. The nucleon $|N,1\rangle$ is then identified as the lowest energy solution with the associated quantum numbers, while the Roper is defined as the next state ($|N,2\rangle$) in the same spin – isospin channel. Turning to the quantum numbers of the Λ provides not only the energy $E_{\Lambda,1}$ and wave–function $|\Lambda,1\rangle$ of this hyperon but also the analogous quantities for the radially excited Λ's: $E_{\Lambda,m}$ and $|\Lambda,m\rangle$ with $m\ge 2$. These calculations are repeated for the other spin – isospin channels yielding the spectrum not only of the ground state $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons but also their radial excitations. Of course, flavor symmetry breaking couples all possible $SU(3)$ representations. When diagonalizing (2.12) we consider the basis built by the representations **8**, $\overline{\mathbf{10}}$, **27**, $\overline{\mathbf{35}}$, **64**, $\overline{\mathbf{81}}$, **125**, $\overline{\mathbf{154}}$ for the $\frac{1}{2}^+$ baryons and **10**, **27**, **35**, $\overline{\mathbf{35}}$, **64**, $\overline{\mathbf{28}}$, **81**, $\overline{\mathbf{81}}$ **125**, $\overline{\mathbf{80}}$ **154**, $\overline{\mathbf{254}}$ for the $\frac{3}{2}^+$ baryons. For the breathing degree of freedom we include basis states which are up to 4GeV above the ground states of the flavor symmetric piece (2.11), *i.e.* $|\mathbf{8},1\rangle$ and $|\mathbf{10},1\rangle$ for the $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons, respectively. This seems to be sufficient to get acceptable convergence when diagonalizing (2.12). It should be noted that not all of the above SU(3) representations appear in each isospin channel. For example, there is no Λ–type state in the $\overline{\mathbf{10}}$.

In Table 1 the predictions for the mass differences[5] with respect to the nucleon of the eigenstates are shown for two values of the Skyrme parameter e. The agreement with the experimental data is quite astonishing, not only for the ground state but also for the radial excitations. Only the prediction for the Roper resonance ($|N,2\rangle$) is on the low side. This is common to the breathing mode approach [13,14]. As far as data are available the other first excited states are quite well reproduced. On the other hand for the $\frac{1}{2}^+$ baryons the energy eigenvalues for the second excitations overestimate the corresponding empirical data somewhat. However, the pattern $M(|N,2\rangle) < M(|\Sigma,2\rangle) < M(|\Lambda,2\rangle)$ is reproduced. The predicted Σ and Λ type states with $m=2$ are about 200MeV too high. For the $\frac{3}{2}^+$ baryons with $m=2$ the agreement with data is much better, on the 3% level. On the whole, the present model gives fair agreement with the available data. This certainly supports the picture of coupled radial and rotational modes.

Above we stated that the factor $e^{3\sigma}$ in (2.4) mitigated the effects of flavor symmetry breaking in the soliton sector. For the soliton solution the σ field is always negative. Hence the contribution of the mass–type symmetry breaker to $s(x)$ in (2.10) is significantly reduced as compared to the pure pseudoscalar case. As already discussed, the mass–type symmetry breaker strongly dominates over the kinetic–type, which is suppressed by the factor $e^{2\sigma}$. Since applying the breathing mode approach to the pure pseudoscalar model (*i.e.* $\sigma\equiv 0$) overestimates the flavor symmetry breaking in the baryon mass differences [16],

[4] The Clebsch–Gordon coefficients not provided in [28] are numerically computed as described in footnote 14 of [16] based on the Euler angle decomposition of [6]

[5] In soliton models commonly only mass differences are considered to avoid the inclusion of meson loop corrections which reduce the absolute values substantially [30]

Table 2. The strange content fractions X_S for the $J = 1/2$ ground state compared to the flavor symmetric case (pure octet). Up to the provided precision the results coincide for $e = 5.0$ and $e = 5.5$. The differences to the pure octet case indicate the significance of the higher dimensional representations

	N	Λ	Σ	Ξ
$e = 5.0$	0.16	0.25	0.30	0.37
Octet	0.23	0.30	0.37	0.40

the incorporation of the scalar glueball field improves on the predicted baryon spectrum [17]. In the pure Skyrme model a reduction of symmetry breaking effects can be gained by decreasing the Skyrme constant e. Unfortunately this also lowers the difference between the nucleon and the Δ masses. As can be observed from Table 1 in [16] an overall satisfactory picture cannot be obtained in the breathing mode approach to the pure pseudoscalar model.

In Fig. 1 the dominant pieces of the radial wave–functions

$$f_\mu^{(B,m)}(x) = \sum_{n_\mu} C_{\mu,n_\mu}^{(B,m)} f_{\mu,n_\mu}^{(0)}(x) \qquad (2.14)$$

are shown for the $\frac{1}{2}^+$ ground states. Here $f_{\mu,n_\mu}^{(0)}(x)$ denote the radial eigenfunctions of the flavor symmetric formulation (2.11) multiplied by $(\alpha^3(x)\beta^4(x)/m(x))^{(1/4)}$ [16]. It should be noted that these radial wave–functions are normalized with respect to a metric $m(x)$ which is singular at $x = 0$, *cf.* (2.10). Hence all wave–functions vanish at that particular point. We observe that these ground states are dominated by the radial ground state in the octet representation. Nevertheless the contributions from the higher dimensional representations are not negligible either. This can also be seen from the strange content fractions of these baryons. This quantity can be associated with the matrix elements $X_S = \langle(1 - D_{88})/3\rangle$ [31]. The sizable deviations from the pure octet results are shown in Table 2. It is also interesting to note that the strange content fraction for the Roper and the $N(1710)$ respectively decrease from 23% to 14% and from 25% to 19% ($e = 5.0$) due to flavor symmetry breaking.

In Fig. 2 the dependence on the scaling variable for the first two excited nucleon states is shown. As expected the Roper is dominantly a radial excitation of the octet. However, there are also sizable contributions of the radial ground states in the higher dimensional representations. For the state which we want to associate with the $N(1710)$ we indeed find that the $\overline{\mathbf{10}}$ contributes the major share. On the other hand the admixture of the $2\hbar\omega$ excitation of the octet is not negligible either. Hence the identification of the $N(1710)$ with $|N, \overline{\mathbf{10}}\rangle$ appears as an over–simplification.

It should be noted that the model predicts yet another eigenstate of (2.12) just about 50MeV above the state we just identified as $N(1710)$. This is essentially the linear combination of the $|N, \overline{\mathbf{10}}\rangle$ and the $2\hbar\omega$ radial excitation of the octet which is orthogonal to the wave–function shown in the right panel of Fig. 2. Although the Particle Data Group (PDG) [29] gives 2.1GeV for the average value of the mass of the third resonance in the $P11$ channel there is also an analysis [32] of the data which yields a significant lower resonance position, 1.885 ± 0.030GeV. In particularly one should note that the four–pole fit of [33] predicts two states around 1.75GeV in the $P11$ channel which are less than 10MeV apart. We find a similar scenario on the Σ channel, although about 200MeV too high. In Table 1 the $|\Sigma, m = 2\rangle$ state has been considered to be the $\Sigma(1770)\,P11$. In addition we observe a Σ–type state 1.135GeV above the nucleon for $e = 5.0$ and 1.181GeV for $e = 5.5$. Eventually this could be identified with the $\Sigma(1880)\,P11$ [29][6]. One should bear in mind that the analyses leading to this two–star resonance are somewhat dated and are spread between[7] 1.826 ± 0.020 [34] and 1.985 ± 0.050GeV [35]. Nevertheless one is inclined to consider the predicted two almost degenerate states in that energy regime as a nice feature of the present model. In the Λ and Ξ channels no such doubling is observed as the $\overline{\mathbf{10}}$ does not contain states with the quantum numbers of these baryons. However, this representation contains a $Y = -1$, $I = \frac{3}{2}$ state which is not considered here.

States with the quantum numbers of the Z^+ exist besides in the $\overline{\mathbf{10}}$ also in the $\overline{\mathbf{35}}$, $\overline{\mathbf{81}}$ and $\overline{\mathbf{154}}$ representations. Actually these are always the complex conjugates of representations which also contain Ω–type states. This is a direct consequence of the complex conjugation being equivalent to a reflection at the $Y = 0$ axis. Upon this reflection the Ξ^*–type state is transformed into a nucleon type state which then satisfies the conditions $J = I$ and $S = 0$ while the Ω becomes the Z^+.

The resulting radial structure of the Z^+ wave–function is displayed in the left panel of Fig. 3. We recognize that the higher dimensional contributions are not negligible. In particular the amplitude of the $\overline{\mathbf{35}}$ is almost half as large as the leading order piece residing in the $\overline{\mathbf{10}}$. It should be noted that up to only first order in flavor symmetry breaking the Z^+ states in $\overline{\mathbf{10}}$ and $\overline{\mathbf{35}}$ have non–vanishing overlap. In the estimate of [3] this overlap was not taken into account. As the second order perturbation to the energy of a ground state is always negative one would speculate that the mass of the Z^+ would even be reduced when this effect was included. However, here the mass of the Z^+ is predicted to be 1.57GeV and 1.59GeV for $e = 5.0$ and $e = 5.5$, respectively. This is 40 to 60MeV larger than the result of [3]. As compared to the octet, the centrifugal barrier for states in the $\overline{\mathbf{10}}$ representation is stronger. Hence the corresponding eigenfunctions of (2.11) are localized at larger values of the scaling variable x. This effect can be observed by comparing the radial wave–functions in Figs. 1 and 3. In turn it leads to more sizable contributions associated with flavor symmetry breaking (2.12) than in the model without breathing mode. In order to further illuminate the statement that the Z^+ wave–function is pushed to larger values of x also the wave–function of the Δ is shown in Fig. 3. We note that although in lead-

[6] In [3] this state was speculated to be the pure $|\Sigma, \overline{\mathbf{10}}\rangle$

[7] *Cf.* the references compiled by the PDG: p. 652 in [29]

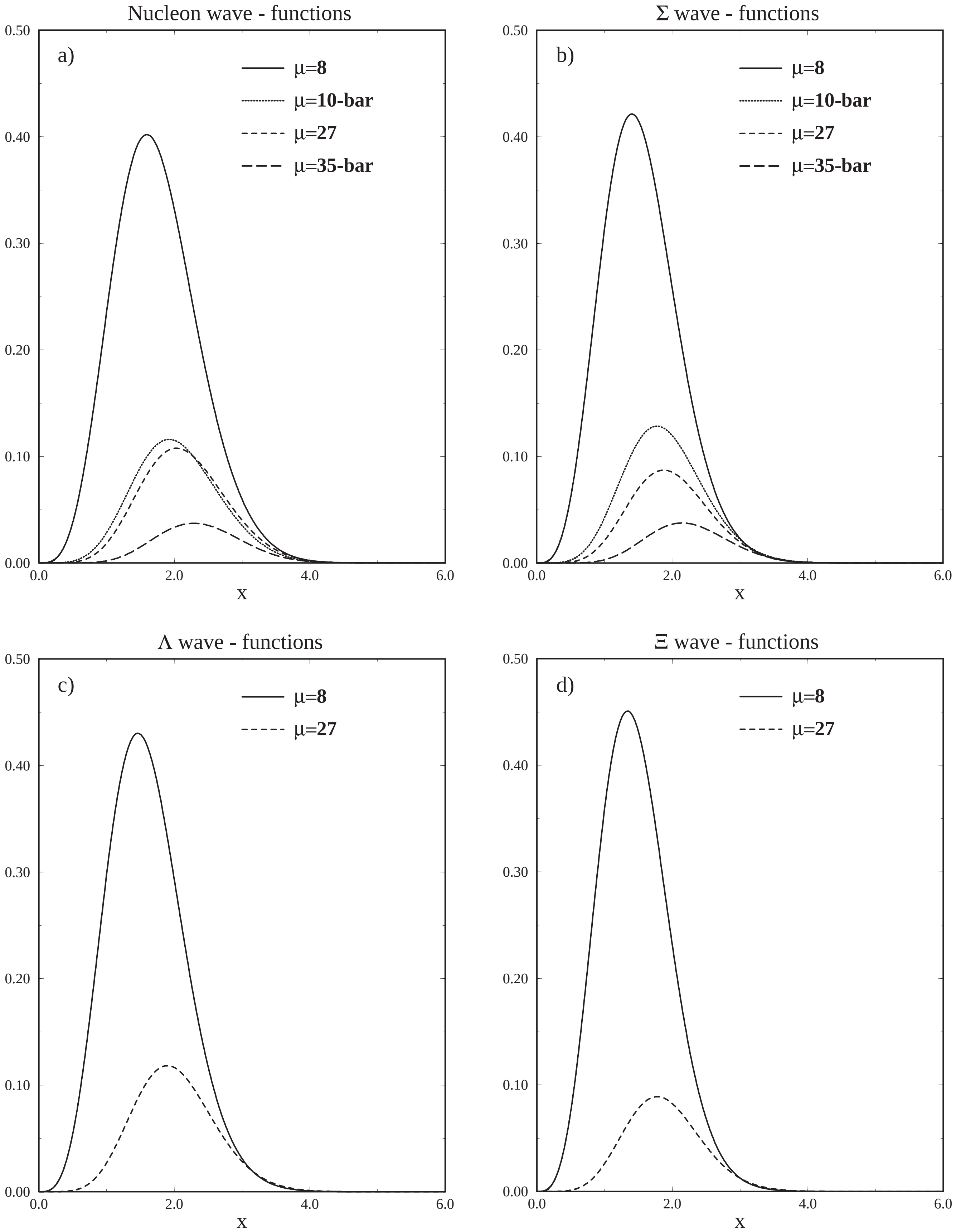


Fig. 1. The contributions of the lowest SU(3) representations μ to the radial parts of the ground state baryons with $J = 1/2$. Here $e = 5.0$ is used

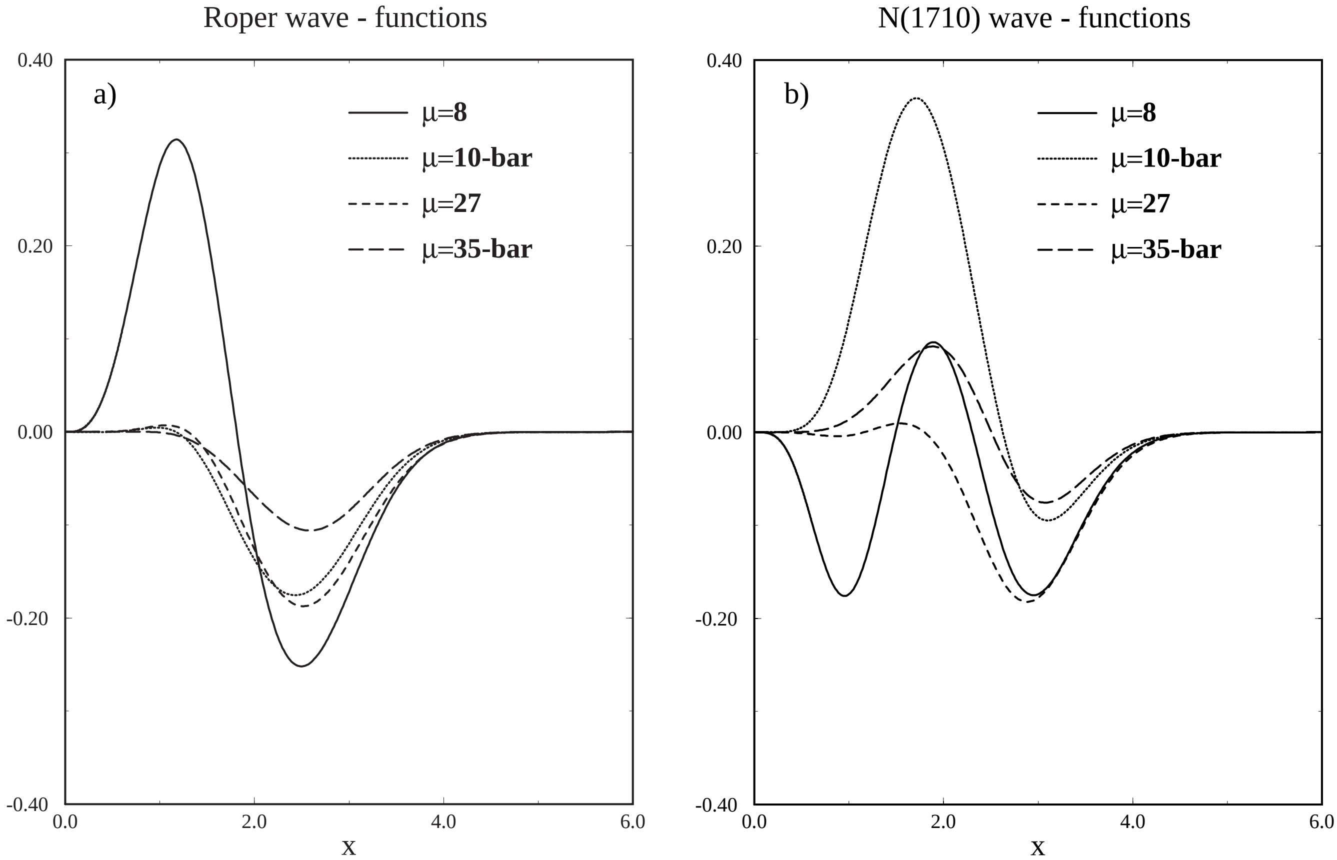


Fig. 2. The contributions of the lowest SU(3) representations μ to the excited states with nucleon quantum numbers. Here $e = 5.0$ is considered

ing order the Z^+ and Δ have the same Casimir eigenvalue $C_2(\overline{\mathbf{10}}) = C_2(\mathbf{10}) = 6$ the centrifugal barrier is smaller for the Δ because $\alpha(x) > \beta(x)$ in the Hamiltonian (2.11). It should be remarked that the singular behavior of the metric $m(x)$, which enters the evaluation of all matrix elements, intensifies this effect. The increase of the mass due to the Z^+ being localized at larger x is a leading order effect in flavor symmetry breaking which effects the strange content fraction X_s. In the flavor symmetric case we find $X_s = 25\%$ for the Z^+. When the flavor symmetry breaking effects are included it is reduced to about 18%. This implies that the Z^+ possesses a significant cloud of non–strange mesons. It is finally worthwhile to note that the first excited state in the Z^+ channel is at 2.02(2.07)GeV for $e = 5.0(5.5)$.

3 Estimate of widths

The soliton model described in the preceding section has been shown to reasonably describe not only the spectrum of the low–lying $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons but also various baryon static properties. In particular the inclusion of the radial collective coordinate properly reproduces the experimentally observed deviation from U–spin symmetry of the predictions for the baryon magnetic moments [16,17]. This deviation remains unobserved as long as flavor symmetry breaking effects on the extension of the soliton are not included [36,5]. Hence one is inclined to assume that also the widths for various decays of the predicted states (resonances) are reasonably described. Here we are interested in the decay $Z^+ \to K^{(+,0)}N$ which should be mediated by a pseudoscalar Yukawa coupling. In soliton models such a coupling is not directly obtainable as to leading order in $1/N_C$ terms linear in the meson fluctuations vanish by definition. One possibility to avoid this problem is to adopt the Goldberger–Treiman relation, which relates the relevant coupling constant to the axial charge. Along that line the matrix element of D_{33} was used in [11] to predict the amplitude for the decay $\Delta \to N\pi$ and relate it to the πN coupling constant. In the present model the situation is even less transparent as we also demand the dependence of the transition operator on the scaling variable x. Assuming, for the time being, that we have obtained the relevant operator in the space of the collective coordinates the corresponding matrix element will yield the coupling constant $G_{B'\to B\phi}$ associated with the decay of the resonance B' to another baryon state B and a meson ϕ. This coupling constant then enters the width via

$$\Gamma(B' \to B\phi) = \frac{3G^2_{B'\to B\phi}}{8\pi M_{B'} M_B} \, |\boldsymbol{p}_\phi|^3 \ . \tag{3.1}$$

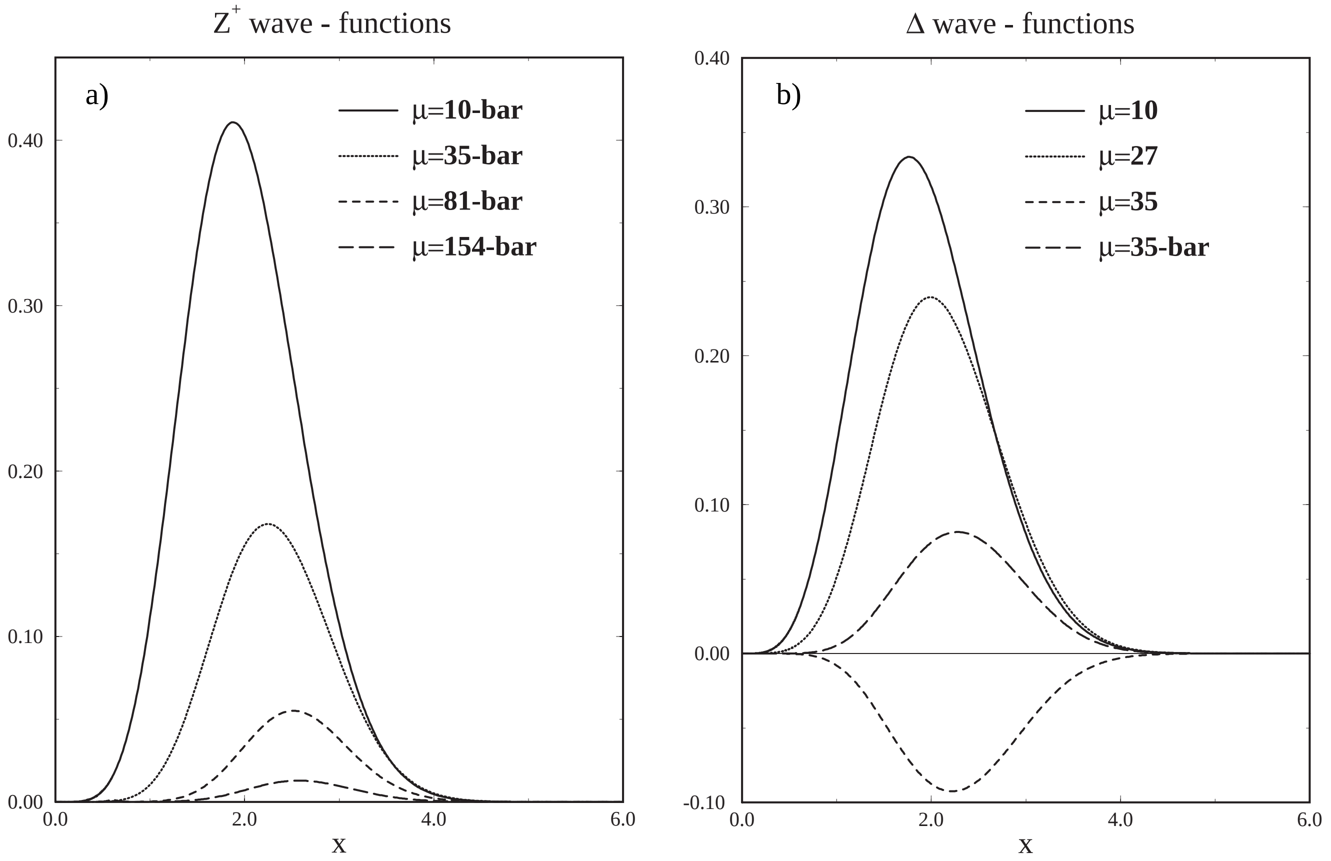


Fig. 3. The contributions of the lowest SU(3) representations μ to the penta–quark baryon Z^+. Also the wave–functions for the Δ–resonance are shown. Here $e = 5.0$ is considered

Here $|\boldsymbol{p}_\phi|$ is the momentum of the outgoing meson in the rest frame of the resonance B'. The cubic dependence on $|\boldsymbol{p}_\phi|$ arises as the amplitude (which enters the width quadratically) of the pseudoscalar Yukawa coupling is linear in $|\boldsymbol{p}_\phi|$. In addition the phase–space provides one power in that momentum. For states which are located just slightly above threshold this cubic dependence on the momentum rather than the value of the coupling constant will be the most crucial ingredient to calculate the width of B'. Hence it is sufficient to get a rough estimate for the coupling constants in order to allow for a comparison of various decay processes. It is therefore suggestive to adopt the following strategy: For the flavor part of the relevant operator we adopt $D_{\phi,3}$ which in leading order $1/N_C$ is the only possible operator compatible with flavor covariance. For the scaling piece we will consider different powers, $\mu^{-n} = x^{2n/3}$. Different values for the power n can be motivated by the long range behavior of the pseudoscalar fields which built up the soliton. Taking straightforwardly the matrix element of the pion field would result in $n = 3$ from the spatial integration. Considering that due to the pseudoscalar character of that field the Fourier transformation involves the spherical Bessel function $j_1(qr)$ would add a factor r to the integrand [37], whence $n = 4$. On the other hand one could argue that in the chiral limit ($m_\pi = 0$) the coupling constant is directly related to the amplitude of the soliton at large r [38]. As the massless pion field decays like $1/r^2$ one would be inclined to adopt $n = 2$. In this way we will obtain at least the generic behavior of the coupling constant while the major ingredient for the decay width, the momentum $|\boldsymbol{p}_\phi|$, is computed from the spectrum calculated in the preceding section. The widths of various decays will finally be compared by adjusting the absolute magnitude to the process $\Delta \to N\pi$, *i.e.* $\Gamma(\Delta \to N\pi) \approx 120$MeV. This corresponds to a multiplicative normalization of the decay constants which is also suggested by large–N_C considerations [39].

To be precise, we will compute matrix elements of the form

$$G_{B'\to B\phi} = C_\Delta \langle B'm'|x^{2n/3} D_{a3}|Bm\rangle, \qquad (3.2)$$

with C_Δ fitted to $\Gamma(\Delta \to N\pi)$. Here we take $a = 3$ and $a = 4 \pm i5$ for strangeness conserving ($\phi = \pi$) and strangeness changing ($\phi = K$) decays, respectively. The latter case is only relevant for the decay of the Z^+. The baryon wave–functions are those of (2.13) which stem from diagonalizing the full collective Hamiltonian. The resulting widths as well as the ratio of the coupling constants between the Δ and the nucleon to the pion are shown in Table 3. Let us recall that as $\Gamma(\Delta \to N\pi)$ is kept fixed one should consider $g_{\pi N\Delta}$ as an input quantity.

Table 3. Decay widths (in MeV) and ratio of πN and $\pi\Delta$ coupling constants using the matrix elements (3.2). The decay width $\Gamma(\Delta \to N\pi) \approx 120$MeV is fixed. R denotes the Roper (1440) resonance. Experimental data are extracted from [29]

	e=5.0			e=5.5			expt.
n	4	3	2	4	3	2	
$\Sigma^* \to \Sigma\pi$	1	1	1	2	2	2	4 ± 1
$\Sigma^* \to \Lambda\pi$	33	38	42	37	38	43	32 ± 4
$\Xi^* \to \Xi\pi$	5	7	10	7	9	11	10 ± 2
$R \to N\pi$	429	281	156	424	260	145	200 to 320
$R \to \Delta\pi$	4	2	2	9	6	3	50 to 80
$Z^+ \to NK$	118	121	124	130	124	126	?
$g_{\pi NN}/g_{\pi N\Delta}$	0.77	0.79	0.83	0.77	0.79	0.83	0.68

Apparently we find that, at least for the widths of ground state baryons, the dependence on the power n is only moderate. The reason is that for these baryons the major contribution to the scaling part of the matrix element stems from the vicinity of $x = 1$. In addition the shape of the wave–functions of these baryons is quite similar in that region, *cf.* Fig. 1. Hence the effect of different n is compensated by normalizing to $\Gamma(\Delta \to N\pi)$. Regarding the crudeness of our estimate the predicted widths for the processes $\Sigma^* \to \Sigma\pi$, $\Sigma^* \to \Lambda\pi$ and $\Xi^* \to \Xi\pi$ as well as the ratio $g_{\pi NN}/g_{\pi N\Delta}$ are in fair agreement with the empirical data. The case of the Roper (1440) resonance is different. Here we recognize a strong dependence on the power n. This is a consequence of the associated breathing mode wave–function having a node around $x = 1.5 - 2.0$, *cf.* Fig. 2. For the decay $R \to N\pi$ the value $n = 3$ appears to be reasonable. However, one should be careful with such a conclusion as the too low prediction for the mass of the Roper might falsify the result for the width. This is even more pronounced for the process $R \to \Delta\pi$. Table 3 indicates that at least one order of magnitude is missing for the width. For the masses given in Table 1 the momentum of the outgoing pion is 66 (91)MeV for $e = 5.0(5.5)$. Substituting the physical momentum, $|\boldsymbol{p}_\pi| = 147$MeV could account for an order of magnitude for the decay width (3.2). Again we recognize that the decay widths are significantly more sensible to the mass parameters than to the decay constants, in particular for processes with kinematics just above threshold. For the decay of the Z^+, the process we are mostly interested in, we recognize neither a strong dependence on the power n nor on the model parameter e. From the results shown in table 3 it seems fair to conclude that the width of the process $Z^+ \to NK$ follows closely the width of $\Delta \to N\pi$.

Rather than just identifying the matrix element of the pseudoscalar field with the coupling constant $G_{B'\to B\phi}$ one could imagine to compute this coupling constant via the axial current and adopt the Goldberger–Treiman relation. In three flavor space this is different from the above approach because an additional operator, whose contribution to $G_{B'\to B\phi}$ is suppressed by $1/N_C$, enters the calculation. In this approach one calculates the matrix elements

Table 4. Decay widths (in MeV) and ratio of πN and $\pi\Delta$ coupling constants using the matrix elements (3.4). The decay width $\Gamma(\Delta \to N\pi) \approx 120$MeV is fixed. R denotes the Roper (1440) resonance. Experimental data are extracted from [29]

	e=5.0	e=5.5	expt.
$\Sigma^* \to \Sigma\pi$	2	3	4 ± 1
$\Sigma^* \to \Lambda\pi$	64	63	32 ± 4
$\Xi^* \to \Xi\pi$	22	23	10 ± 2
$R \to N\pi$	71	71	200 to 320
$R \to \Delta\pi$	2	3	50 to 80
$Z^+ \to NK$	82	83	?
$g_{\pi NN}/g_{\pi N\Delta}$	0.70	0.69	0.68

$$G_{B'\to B\phi} \sim \langle B'm'|\Big\{\Big(g_1 x^{4/3} + g_2\Big) D_{a3} + g_3 \frac{x^{2/3}}{\beta(x)} \sum_{\alpha,\beta=4}^{7} d_{3\alpha\beta} D_{a\alpha} R_\beta\Big\}|Bm\rangle. \quad (3.3)$$

The constants g_1, g_2 and g_3 are functionals of the static soliton and can be extracted from [16,17]. The additional operator involves the right SU(3)–generators R_a. As it is multiplied by the inverse moment of inertia for rotations into strange direction the contribution of this operator to the coupling constant will be suppressed by $1/N_C$. Hence the adjustment of the coupling constants to the decay $\Delta \to N\pi$ in the spirit of the large–N_C expansion [39] requires to only normalize g_1 and g_2 rather than the whole matrix element (3.4). The numerical results for that calculation are shown in Table 4. The operator (3.4) does not seem to be very well suited in particular because the width for the Roper decaying into a nucleon and a pion is significantly underestimated. We note that the dependence on the momentum of the outgoing meson cannot be made responsible for the short–coming in this process as the resonance is far away from threshold. For $e = 5.5$ we have $|\boldsymbol{p}_\pi| = 354$MeV which is not too different from the physical value of 396MeV. Hence using the physical masses would at best give a 40% increase of the width. Also the widths for the decays $\Sigma^* \to \Lambda\pi$

and $\Xi^* \to \Xi\pi$ turn out to be somewhat too large. On the other hand the width for Z^+ is slightly reduced as compared to the use of (3.2). This is mainly due to the fact that the two SU(3) operators in (3.4) interfere destructively for the state $|Z^+, \overline{\mathbf{10}}\rangle$; contrarily they interfere constructively for the ordinary baryons. To be precise, $\langle \Delta, \mathbf{10}|D_{33}|N, \mathbf{8}\rangle = (1/2)\langle \Delta, \mathbf{10}| \sum_{\alpha,\beta=4}^{7} d_{3\alpha\beta} D_{3\alpha} R_\beta |N, \mathbf{8}\rangle$ while $\langle Z^+, \overline{\mathbf{10}}|D_{K3}|N, \mathbf{8}\rangle =$
$-(1/2)\langle Z^+, \overline{\mathbf{10}}| \sum_{\alpha,\beta=4}^{7} d_{3\alpha\beta} D_{K\alpha} R_\beta |N, \mathbf{8}\rangle$.

To summarize this section it seems reasonable to state that the breathing mode approach to the Z^+ predicts a width of that state of the order of 100MeV. This is considerably larger than the prediction of 15MeV found in [3]. As discussed intensively, a major reason for this difference is not the difference in the coupling constant for the process $Z^+ \to KN$ but rather the larger mass found for the Z^+ in the present approach. As compared to [3] the result for the mass of the Z^+ has increased only moderately by 50MeV. Nevertheless it has noticeable consequences for the width of the only possible decay mode of this penta–quark state, $Z^+ \to NK$. The momentum of the outgoing kaon grows from $|\boldsymbol{p}_K| = 254$MeV to 320MeV increasing the width by a factor of two as for processes which are just above threshold the momentum of the outgoing meson is a quickly rising function of the resonance position. In order to further compare the width of the Z^+ with the result of [3] it should be noted that such a comparison should concern the 80MeV displayed in Table 4 because those authors also included the $\sum_{\alpha,\beta=4}^{7} d_{3\alpha\beta} D_{a\alpha} R_\beta$ operator. As discussed this operator lowers the prediction for the width of the Z^+ due to the destructive interference with the leading operator D_{a3}. The moment of inertia for rotations into strange directions ($\beta(x)$) appears in the denominator of the additional operator. Hence the contribution of this operator will be most sensible to the small–x shape of the wave–function. As already discussed, the wave–function for the Z^+ penta–quark is more pronounced at larger values of x due to the angular barrier being stronger for states in the higher dimensional SU(3) representations. As indicated in Fig 3 this is also the case when we compare with the Δ wave–function whose matrix elements set the scale for our estimate of the width. As a result the contribution of the additional SU(3) operator is reduced even further. However, this is only a 20–30% effect and still does not explain the full discrepancy with [3]. At this point one should note that in [3] the numerical results for the widths of the $\frac{3}{2}^+$ baryons are erroneous[8]. As those overestimated widths have subsequently been employed to set the overall scale this is likely to be the reason for the remaining discrepancy.

4 Conclusions

In the present study we have investigated the coupling between radial and (flavor) rotational motion of a chiral soliton in flavor SU(3). Upon canonical quantization of the corresponding collective coordinates this approach not only describes the spectrum of the low–lying $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons but also that of the excited states in the respective channels. Besides mixing of various SU(3) representations the model in particular may account for an eventual resonance doubling [33] in the nucleon P11 channel around 1.75GeV. A similar scenario is observed for the Σ channel. These results provide additional support for identifying baryon states in the $\overline{\mathbf{10}}$ representation of flavor SU(3) with observed resonances. Subsequently this picture leads to the question of properly identifying those baryon states in such higher dimensional representations which do not have counterparts in the octet or decuplet. In this respect the Z^+ ($Y = 2$, $I = 0$, $J^\pi = \frac{1}{2}^+$) is the most interesting candidate as probably being the lightest one. Previously [3] this state was considered to be a pure $\overline{\mathbf{10}}$ baryon. That calculation, however, was not a full model calculation but rather a compilation of possible terms allowed by the flavor symmetries of the model. The constants of proportionality were determined from the known baryon spectrum and radial degrees of freedom were frozen. In this treatment it seems doubtful to *ad hoc* identify of the $N(1710)$ (potentially a radially excited nucleon) with the nucleon state in the $\overline{\mathbf{10}}$ representation. Here we have reflected on that assumption by carrying out a full model calculation and emphasizing on the admixture of both, higher dimensional SU(3) representation as well as radially excited baryon states which otherwise have identical quantum numbers. It should be noted that both types of admixture are mediated through flavor symmetry breaking. Despite these major extensions of the model treatment, the present prediction for the mass of the Z^+, 1.58GeV, is only about 50MeV higher than that of [3] but still about 60MeV lower than the value found in [1]. As discussed, in the soliton model the determination of the coupling constants for various decays bears quite some uncertainties, nevertheless the model calculations suggest $\Gamma(Z^+ \to NK) \sim 100$MeV as an estimate for the width of the Z^+. Quite a substantial uncertainty should be attributed to this value. Comparison of the different estimates collected in Sect. 3 suggests ±30MeV. This should be considered a lower bound for the uncertainty.

Here we have employed a soliton model which besides the pseudoscalar octet mesons contains a scalar field. This scalar field has been introduced as to mock up the QCD trace anomaly. Although this is presumably not the most natural choice for an effective meson theory, we have motivated this model from the simplicity to include the breathing degree of freedom and its previous success to reason-

[8] For example, for the process $\Delta \to N\pi$ the use of eq ([3]:42) together with the empirical values for the masses of the involved hadrons and the suggested coupling constant $G_0 = 19$ yields a width of 64MeV rather than the alleged 110MeV. However, the expression ([3]:56) for the width of the Z^+ has been worked out correctly. As an attempt to locate the possible error it could be remarked that the replacement of the factor M_2/M_1 by its inverse in eq ([3]:49) yields the numerical results presented in eqs ([3]:42)–([3]:45) for the decay widths of the $3/2^+$ baryons. The analogous replacement in eq ([3]:56) results in a Z^+ width of about 40MeV [40]

ably describe the spectrum of the low–lying $\frac{1}{2}^+$ and $\frac{3}{2}^+$ baryons as well as various of their static properties.

Finally one could object that the prediction of *exotic* states like Z^+ would completely be due to the adopted quantization scheme for the flavor degrees of freedom. In the alternative bound state approach [41] a penta–quark state with the quantum numbers of the Z^+ would emerge as a bound system of the soliton and a kaon, while the ordinary hyperons are considered as anti–kaons bound in the soliton background. Such penta–quark states are found to be unbound unless the kaon mass is artificially tuned to about 1GeV. However, the resonance doubling found in the nucleon and Σ channels around 2GeV is not without experimental support which indicates that *exotic* representations like the $\overline{\mathbf{10}}$ indeed have physical significance.

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